학번	-	이름	雀
단원명	-	문항번호	自作-2

문항	연속확률변수 X의 확률밀도함수가 $a \cdot \sqrt[3]{\tan x}$ 일 때, 상수 a의 값을 구하여라. (단, $0 < x < \frac{\pi}{2}$)

<치환>

[1] $t^3 = \tan x$, $x = \tan^{-1}(t^3)$, $dx = \frac{3t^2}{1+t^6}dt$

[2] $a = t^2$, $da = 2t dt$, $a = \tan^{2/3}x$

$$(1)\ \int \sqrt[3]{\tan x}\,dx$$

$$= \int \frac{3t^3}{1+t^6}dt \ \cdots\ [1]$$

$$= \frac{3}{2}\int \frac{a}{1+a^3}da \ \cdots\ [2]$$

$$= \frac{3}{2}\int\left(-\frac{1}{3(a+1)} + \frac{a+1}{3(a^2-a+1)}\right)da$$

$$= \frac{1}{2}\int\left(\frac{2a-1}{2(a^2-a+1)} + \frac{3}{2(a^2-a+1)} - \frac{1}{a+1}\right)da$$

$$= \frac{1}{4}\ln\frac{a^2-a+1}{(a+1)^2} + \frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{2}{\sqrt{3}}\left(a-\frac{1}{2}\right)\right) + C$$

$$= \frac{1}{4}\ln\frac{\tan^{4/3}x - \tan^{2/3}x + 1}{(\tan^{2/3}x+1)^2} + \frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{2}{\sqrt{3}}\left(\tan^{2/3}x - \frac{1}{2}\right)\right) + C.$$

$$(2)\ \int_0^{\frac{\pi}{2}} \sqrt[3]{\tan x}\,dx$$

$$= \left[\frac{1}{4} \ln \frac{\tan^{4/3} x - \tan^{2/3} x + 1}{\left(\tan^{2/3} x + 1\right)^2} + \frac{\sqrt{3}}{2} \tan^{-1} \left(\frac{2}{\sqrt{3}} \left(\tan^{2/3} x - \frac{1}{2} \right) \right) \right]_0^{\frac{\pi}{2}}$$

$$= \lim_{x \to \frac{\pi}{2}} \left\{ \frac{1}{4} \ln \frac{\tan^{4/3} x - \tan^{2/3} x + 1}{\left(\tan^{2/3} x + 1\right)^2} + \frac{\sqrt{3}}{2} \tan^{-1} \left(\frac{2}{\sqrt{3}} \left(\tan^{2/3} x - \frac{1}{2} \right) \right) \right\} - \frac{\sqrt{3}}{2} \tan^{-1} \left(-\frac{1}{\sqrt{3}} \right)$$

$$= \lim_{t \to \infty} \left\{ \frac{1}{4} \ln \frac{t^{4/3} - t^{2/3} + 1}{\left(t^{2/3} + 1\right)^2} + \frac{\sqrt{3}}{2} \tan^{-1} \left(\frac{2}{\sqrt{3}} \left(t^{2/3} - \frac{1}{2} \right) \right) \right\} + \frac{\sqrt{3}}{12} \pi$$

$$= \frac{1}{4} \ln 1 + \frac{\sqrt{3}}{4} \pi + \frac{\sqrt{3}}{12} \pi$$

$$= \frac{\pi}{\sqrt{3}}.$$

$\int_0^{\frac{\pi}{2}} a \cdot \sqrt[3]{\tan x}\, dx = \frac{\pi}{\sqrt{3}} a = 1$에서

$$\therefore a = \frac{\sqrt{3}}{\pi}$$