

\* 2019년 10월 시행 교육청 고3 수학 나형 30번.

$$a > 0, f(x) = x^2 + \dots, g(x) = x^3 + \dots$$

$$(가) f(0) = g(0)$$

$$(4) \lim_{x \rightarrow 0} \frac{f(x)}{x} = 0, \lim_{x \rightarrow a} \frac{g(x)}{x-a} = 0$$

$$(4) \int_0^a \{g(x) - f(x)\} dx = 36$$

$$f(0) = 0, f(x) = x \times f_1(x), f_1(0) = 0$$

$$\therefore f(x) = x \times x = x^2 \quad (2차함수)$$

$$g(x) = (x-a)g_1(x), g_1(a) = 0$$

$$\therefore g(x) = (x-a)(x-a)x \quad (\because f(0) = g(0) = 0)$$

$$= x^3 - 2ax^2 + a^2x$$

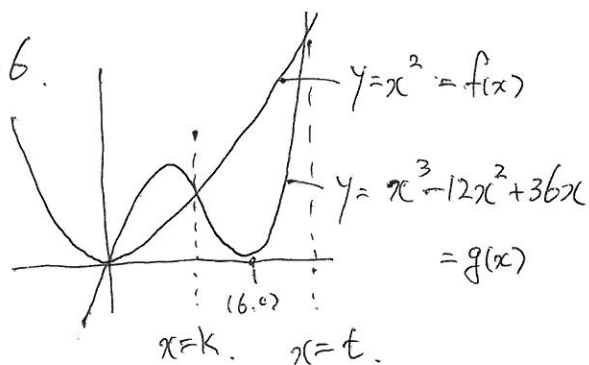
$$\therefore \int_0^a \{g(x) - f(x)\} dx$$

$$= \int_0^a \{x^3 - (2a+1)x^2 + a^2x\} dx = \left[ \frac{x^4}{4} - \frac{(2a+1)x^3}{3} + \frac{a^2x^2}{2} \right]_0^a$$

$$= \frac{a^4}{4} - \frac{2a^4}{3} - \frac{a^3}{3} + \frac{a^4}{2} = \frac{3-8+6}{12}a^4 - \frac{a^3}{3} = \frac{a^4}{12} - \frac{a^3}{3} = 36$$

$$\therefore a^4 - 4a^3 - 36 \times 12 = 0 \rightarrow a^3(a-4) = 36 \times 12 = 36 \times 6 \times 2 = 6^3 \times (6-2)$$

$$\therefore a = 6$$



$$x^3 - 13x^2 + 36x$$

$$= x(x^2 - 13x + 36) = 0,$$

$$\{0, k, t\}$$

$$\therefore k=4, t=9. \quad (k < t). \quad \text{따라서} \int_0^6 |f(x) - g(x)| dx \leq$$

$$\int_0^4 \{x^3 - 13x^2 + 36x\} dx + \int_4^6 \{-x^3 + 13x^2 - 36x\} dx$$

$$= \left[ \frac{x^4}{4} - \frac{13x^3}{3} + 18x^2 \right]_0^4 - \left[ \frac{x^4}{4} - \frac{13x^3}{3} + 18x^2 \right]_4^6 = 2 \times \left( \frac{4^4}{4} - \frac{13 \times 4^3}{3} + 18 \times 4^2 \right) - \left( \frac{6^4}{4} - \frac{13 \times 6^3}{3} + 18 \times 6^2 \right)$$

$$= 2 \times 4^2 \times \left( \frac{12-52+54}{3} \right) - 6^2 \times (9-26+18) = \frac{2 \times 16 \times 14}{3} - 36$$

$$\therefore 3 \int_0^a |f(x) - g(x)| dx = 2 \times 16 \times 14 - 36 \times 3 = 448 - 108 = 340 //$$