가형 30 $f(x) = \ln(e^x+1) + 2e^x$

$h(x) = |g(x) - f(x-k)|$ ① $x=k$에서 최솟값 $g(k)$

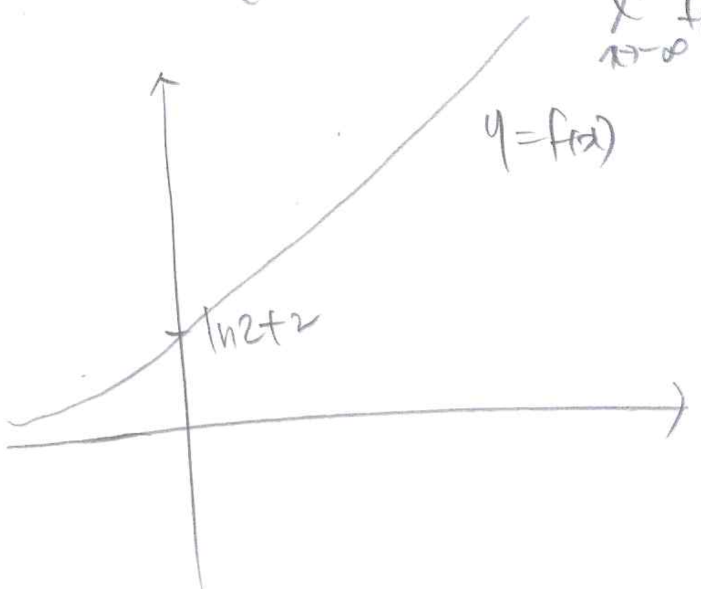
② $[k-1, k+1]$ 최댓값 $2e + \ln\left(\frac{1+e}{\sqrt{2}}\right)$

→ $h(x)$는 최대랑 최소는 가진다는 것을 이용하기 위해 $f(x)$ 그래프를 그려본다.

$f(x) = \ln(e^x+1) + 2e^x$

$f'(x) = \frac{e^x}{e^x+1} + 2e^x > 0$ $f(x)$는 증가함수.

$\lim_{x\to-\infty} f(x) = 0$ $\lim_{x\to\infty} f(x) = \infty$



① $x=k$에서 최솟값 $g(k)$를 가진다.

$|g(k) - f(0)| = g(k)$

→ $g(k) - f(0) \geq 0$ 이면 $g(k) - f(0) = g(k)$

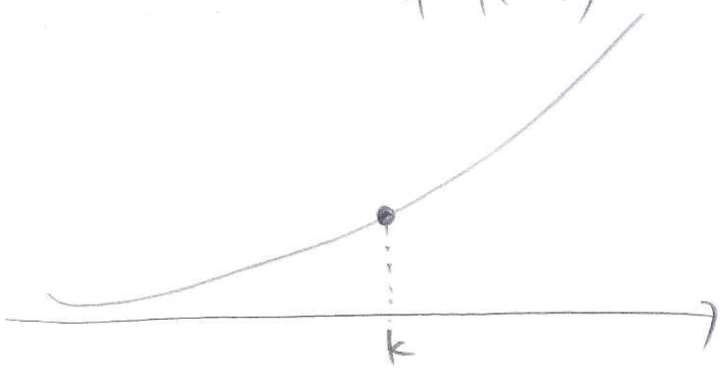
$f(0) = 0$ 인데

$f(0) = \ln 2 + 2$ 니까 모순

$\therefore -g(k) + f(0) = g(k)$ 이고 $f(0) = 2g(k)$ //

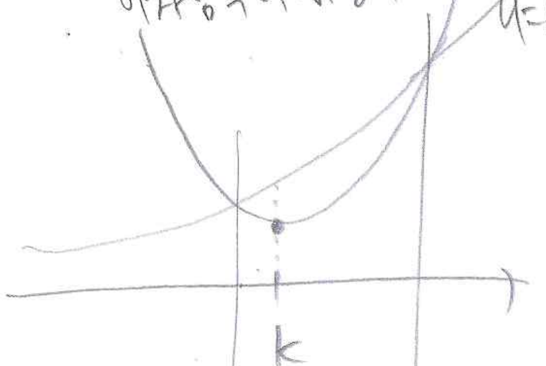
$g(k) < f(0)$ //

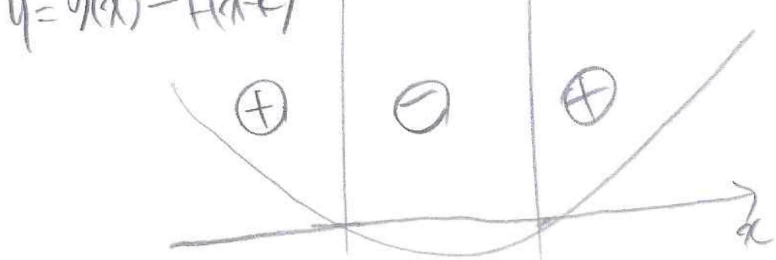
② 그래프 안그림



g(k)는 f(0)보다 작아야 한다.

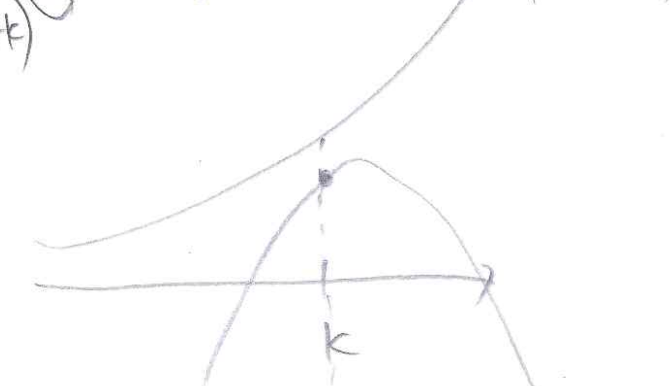
① 이차함수 아래로볼록이면



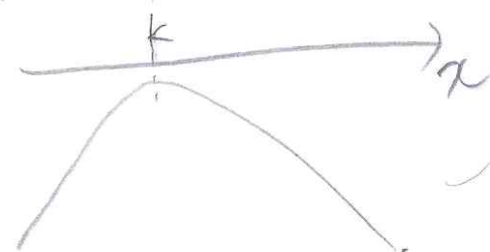


y=g(x)−f(x−k)

② 이차함수 위로볼록이면



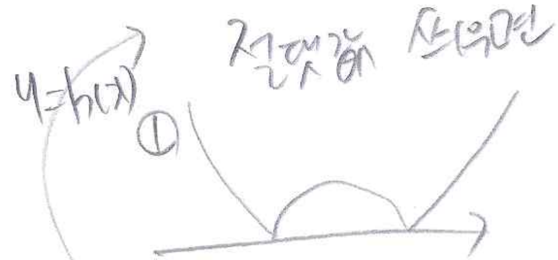
$y=g(x)-f(x-k)<0$



→ k에서 극대가짐 $g'(k)-f'(0)=0$

$\boxed{g'(k)=\frac{e}{2}}$

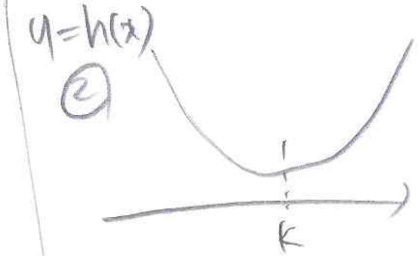
절댓값 씌우면



↑ 최솟값이 아님

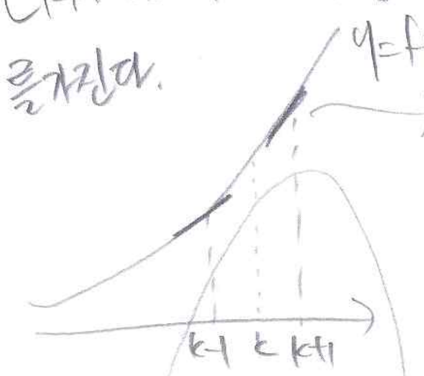
$g(k)=\frac{f(0)}{2}$인데

f(0)=0이 아니니까 모순임



→ 이경우니 맞음

[k−1, k+1]에서 최댓값 $2e+\ln\left(\frac{1+e}{\sqrt{2}}\right)$ 를 가진다.



→ 증가량이 k+1일때 더 크니까 k+1에서 최대를 가질것이라고 추론가능

$\therefore h(k+1)=f(1)-g(k+1)$

$=2e+\ln\left(\frac{1+e}{\sqrt{2}}\right)$

정리

① $g(k) = \frac{f(0)}{2} = \frac{\ln 2 + 2}{2} = \ln\sqrt{2} + 1$

② $g'(k) = \frac{5}{2}$

③ $h(k+1) = f(1) - g(k+1) = 2e + \ln\left(\frac{1+e}{\sqrt{2}}\right)$

$g(x) = ax^2 + bx + c$

① $ak^2 + bk + c = \ln\sqrt{2} + 1$

② $2ak + b = \frac{5}{2}$

③ $\ln(e+1) + \cancel{2e} - g(k+1) = \cancel{2e} + \ln\left(\frac{1+e}{\sqrt{2}}\right)$

$-g(k+1) = \ln\frac{1}{\sqrt{2}}$

$g(k+1) = \ln\sqrt{2}$

$a(k+1)^2 + b(k+1) + c = \ln\sqrt{2}$

$ak^2 + 2ak + a + bk + b + c = \ln\sqrt{2}$

$\ln\sqrt{2} + 1 + \frac{5}{2} + a = \ln\sqrt{2}$

$a = -\frac{7}{2}$

$g'(k - \frac{1}{2}) = 2a(k - \frac{1}{2}) + b$

$= 2ak - a + b$

$= 2ak + b - a$

$= \frac{5}{2} + \frac{7}{2} = 6$

답) 6